Scoring, term weighting, the vector space model

December, 2009
Overview

1. Term frequency

2. tf-idf weighting

3. The vector space
Outline

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2. tf-idf weighting
3. The vector space
Ranked retrieval

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- Also good for applications: Applications can easily consume 1000s of results.
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- Most users are not capable of writing Boolean queries (or they are, but they think it’s too much work).
- Most users don’t want to wade through 1000s of results.
- This is particularly true of web search.
Problem with Boolean search: Feast or famine

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- With a ranked list of documents it does not matter how large the retrieved set is.
We wish to return in order the documents most likely to be useful to the searcher.
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Assign a score – say in $[0, 1]$ – to each document.
Scoring as the basis of ranked retrieval

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- How can we rank-order the documents in the collection with respect to a query?
- Assign a score — say in [0, 1] — to each document
- This score measures how well document and query “match”.
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Let's start with a one-term query.
Query-document matching scores

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- If the query term does not occur in the document: score should be 0.
- The more frequent the query term in the document, the higher the score.
- We will look at a number of alternatives for doing this.
Take 1: Jaccard coefficient

- Recall from IIR 3: A commonly used measure of overlap of two sets
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Recall from IR 3: A commonly used measure of overlap of two sets

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Jaccard coefficient:

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- $A$ and $B$ don’t have to be the same size.
- Always assigns a number between 0 and 1.
Jaccard coefficient: Example

What is the query-document match score that the Jaccard coefficient computes for:
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- Later in this lecture, we’ll use $|A \cap B|/\sqrt{|A \cup B|}$ (cosine) ...
- ... instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.
Recall: Binary incidence matrix

|         | Anthony and Caesar | The Tempest | Hamlet | Othello | Macbeth | ...
|---------|--------------------|-------------|--------|---------|---------|-------
| Anthony | 1                  | 1           | 0      | 0       | 0       | 1     |
| Brutus  | 1                  | 1           | 0      | 1       | 0       | 0     |
| Caesar  | 1                  | 1           | 0      | 1       | 1       | 1     |
| Calpurnia | 0                 | 1           | 0      | 0       | 0       | 0     |
| Cleopatra | 1                | 0           | 0      | 0       | 0       | 0     |
| mercy   | 1                  | 0           | 1      | 1       | 1       | 1     |
| worser  | 1                  | 0           | 1      | 1       | 1       | 0     |

Each document is represented by a binary vector $\in \{0, 1\}^{|V|}$. 
Recall: Binary incidence matrix

<table>
<thead>
<tr>
<th></th>
<th>Anthony</th>
<th>Julius Caesar</th>
<th>The Tempest</th>
<th>Hamlet</th>
<th>Othello</th>
<th>Macbeth</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anthony and Cleopatra</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Brutus</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Calpurnia</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cleopatra</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>mercy</td>
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<tr>
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From now on, we will use the frequencies of terms

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<tr>
<td>Anthony</td>
<td>157</td>
<td>73</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Brutus</td>
<td>4</td>
<td>157</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Caesar</td>
<td>232</td>
<td>227</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Calpurnia</td>
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<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cleopatra</td>
<td>57</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>mercy</td>
<td>2</td>
<td>0</td>
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For now: bag of words model
Term frequency tf

- The term frequency $t_f_{t,d}$ of term $t$ in document $d$ is defined as the number of times that $t$ occurs in $d$. 

We want to use tf when computing query-document match scores. But how? Raw term frequency is not what we want. A document with 10 occurrences of the term is more relevant than a document with one occurrence of the term. But not 10 times more relevant. Relevance does not increase proportionally with term frequency.
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$$w_{t,d} = \begin{cases} 
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- The score is 0 if none of the query terms is present in the document.
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- We define the idf weight of term $t$ as follows:

$$idf_t = \log_{10} \frac{N}{df_t}$$
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- We use \( \log \frac{N}{\text{df}_t} \) instead of \( \frac{N}{\text{df}_t} \) to “dampen” the effect of idf.
- So we use the log transformation for both term frequency and document frequency.
## Examples for idf

Compute $idf_t$ using the formula: 

$$idf_t = \log_{10} \frac{1,000,000}{df_t}$$

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<tr>
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<td></td>
</tr>
<tr>
<td>fly</td>
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Effect of idf on ranking

- idf affects the ranking of documents only if the query has at least two terms.
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- Questions about idf?
### Collection frequency vs. Document frequency

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- This example suggests that df is better for weighting that cf.
The tf-idf weight of a term is the product of its tf weight and its idf weight.
tf-idf weighting

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\[ w_{t,d} = (1 + \log tf_{t,d}) \cdot \log \frac{N}{df_t} \]
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- Best known weighting scheme in information retrieval
- Note: the “-” in tf-idf is a hyphen, not a minus sign!
- Alternative names: tf.idf, tf x idf
Summary: tf-idf

- Assign a tf-idf weight for each term $t$ in each document $d$:
  
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- Increases with the rarity of the term in the collection
### Term, collection and document frequency

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- **Relationship between $tf$ and $cf$?**
Outline

1. Term frequency
2. tf-idf weighting
3. The vector space
### Binary → count → weight matrix

<table>
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<th>Anthony and Caesar</th>
<th>The Tempest</th>
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<tr>
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Each document is now represented by a real-valued vector of tf-idf weights \( \in \mathbb{R}^{|V|} \).
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- This is a very sparse vector - most entries are zero.
Queries as vectors

- Key idea 1: do the same for queries: represent them as vectors in the space
Queries as vectors

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- Key idea 2: Rank documents according to their proximity to the query
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Queries as vectors

- Key idea 1: do the same for queries: represent them as vectors in the space.
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- Proximity = similarity.
- Proximity $\approx$ negative distance.
- Recall: We’re doing this because we want to get away from the you’re-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents.
How do we formalize vector space similarity?

- First cut: distance between two points
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- . . .because Euclidean distance is large for vectors of different lengths.
Why distance is a bad idea

The Euclidean distance of vector $\vec{q}$ and $\vec{d}_2$ is large although the distribution of terms in the query $q$ and the distribution of terms in the document $d_2$ are very similar.
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Questions about basic vector space setup?
Use angle instead of distance

- Rank documents according to angle with query
Use angle instead of distance

- Rank documents according to angle with query
- Thought experiment: take a document $d$ and append it to itself. Call this document $d'$. 

Semantically $d$ and $d'$ have the same content. The angle between the two documents is 0, corresponding to maximal similarity. The Euclidean distance between the two documents can be quite large.
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  - Rank documents according to \( \cosine(query, document) \) in increasing order
- Cosine is a monotonically decreasing function of the angle for the interval \([0^\circ, 180^\circ]\)
Cosine

The figure shows a cosine function graph with the x-axis ranging from 0 to 360 degrees and the y-axis ranging from -1 to 1. The graph depicts a repeating pattern, characteristic of the cosine function, which oscillates between -1 and 1.
What about angles $> 180^\circ$?
Length normalization

- How do we compute the cosine?

A vector can be (length-) normalized by dividing each of its components by its length. Here we use the $L^2$ norm:

$$||x||_2 = \sqrt{\sum_i x_i^2}$$

This maps vectors onto the unit sphere. . . since after normalization:

$$||x||_2 = \sqrt{\sum_i x_i^2} = 1.0$$

As a result, longer documents and shorter documents have weights of the same order of magnitude.
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- As a result, longer documents and shorter documents have weights of the same order of magnitude.
- Effect on the two documents $d$ and $d'$ ($d$ appended to itself) from earlier slide: they have identical vectors after length-normalization.
Cosine similarity between query and document

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\cos(\vec{q}, \vec{d}) = \text{sim}(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| |\vec{d}|} = \frac{\sum_{i=1}^{V} q_i d_i}{\sqrt{\sum_{i=1}^{V} q_i^2} \sqrt{\sum_{i=1}^{V} d_i^2}}
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- \(q_i\) is the tf-idf weight of term \(i\) in the query.
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Cosine similarity between query and document

\[
\cos(\vec{q}, \vec{d}) = \text{sim}(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| \cdot |\vec{d}|} = \frac{\sum_{i=1}^{V} q_i d_i}{\sqrt{\sum_{i=1}^{V} q_i^2} \cdot \sqrt{\sum_{i=1}^{V} d_i^2}}
\]

- \(q_i\) is the tf-idf weight of term \(i\) in the query.
- \(d_i\) is the tf-idf weight of term \(i\) in the document.
- \(|\vec{q}|\) and \(|\vec{d}|\) are the lengths of \(\vec{q}\) and \(\vec{d}\).
Cosine similarity between query and document

\[ \cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| |\vec{d}|} = \frac{\sum_{i=1}^{V} q_i d_i}{\sqrt{\sum_{i=1}^{V} q_i^2} \sqrt{\sum_{i=1}^{V} d_i^2}} \]

- \( q_i \) is the tf-idf weight of term \( i \) in the query.
- \( d_i \) is the tf-idf weight of term \( i \) in the document.
- \(|\vec{q}|\) and \(|\vec{d}|\) are the lengths of \( \vec{q} \) and \( \vec{d} \).
- This is the cosine similarity of \( \vec{q} \) and \( \vec{d} \) or, equivalently, the cosine of the angle between \( \vec{q} \) and \( \vec{d} \).
Cosine for normalized vectors

- For normalized vectors, the cosine is equivalent to the dot product or scalar product.
Cosine for normalized vectors

- For normalized vectors, the cosine is equivalent to the dot product or scalar product.

\[
\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_i q_i \cdot d_i \quad \text{(if } \vec{q} \text{ and } \vec{d} \text{ are length-normalized).}
\]
Cosine similarity illustrated
Cosine: Example

How similar are the novels? *SaS*: Sense and Sensibility, *PaP*: Pride and Prejudice, and *WH*: Wuthering Heights?
How similar are the novels? SaS: Sense and Sensibility, PaP: Pride and Prejudice, and WH: Wuthering Heights?

<table>
<thead>
<tr>
<th>term</th>
<th>SaS</th>
<th>PaP</th>
<th>WH</th>
</tr>
</thead>
<tbody>
<tr>
<td>affection</td>
<td>115</td>
<td>58</td>
<td>20</td>
</tr>
<tr>
<td>jealous</td>
<td>10</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>gossip</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>wuthering</td>
<td>0</td>
<td>0</td>
<td>38</td>
</tr>
</tbody>
</table>

How similar are the novels?
Cosine: Example

term frequencies (counts)

<table>
<thead>
<tr>
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### Cosine: Example

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**log frequency weighting**

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<tbody>
<tr>
<td>affection</td>
<td>3.06</td>
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### Cosine: Example

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(To simplify this example, we don’t do idf weighting.)
Cosine: Example

log frequency weighting

term | SaS | PaP | WH
---|---|---|---
affection | 3.06 | 2.76 | 2.30
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### Cosine: Example

#### log frequency weighting

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#### log frequency weighting & cosine normalization

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<tr>
<th>term</th>
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</tr>
</thead>
<tbody>
<tr>
<td>affection</td>
<td>0.789</td>
<td>0.832</td>
<td>0.524</td>
</tr>
<tr>
<td>jealous</td>
<td>0.515</td>
<td>0.555</td>
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<tr>
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### Cosine: Example

<table>
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<th>log frequency weighting</th>
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\[
\cos(SaS, PaP) \approx 0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx 0.94.
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- \( \cos(SaS, WH) \approx 0.79 \)
**Cosine: Example**

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- \( \cos(SaS, PaP) \approx 0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx 0.94. \)
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- \( \cos(PaP, WH) \approx 0.69 \)
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log frequency weighting

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log frequency weighting & cosine normalization

- \( \cos(SaS, PaP) \approx 0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx 0.94 \).
- \( \cos(SaS, WH) \approx 0.79 \)
- \( \cos(PaP, WH) \approx 0.69 \)
- Why do we have \( \cos(SaS, PaP) > \cos(SaS, WH) \)?
Computing the cosine score
Components of tf-idf weighting

<table>
<thead>
<tr>
<th>Term frequency</th>
<th>Document frequency</th>
<th>Normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>n (natural)</td>
<td>n (no)</td>
<td>n (none)</td>
</tr>
<tr>
<td>l (logarithm)</td>
<td>t (idf)</td>
<td>c (cosine)</td>
</tr>
<tr>
<td>a (augmented)</td>
<td>p (prob idf)</td>
<td>u (pivoted unique)</td>
</tr>
<tr>
<td>b (boolean)</td>
<td></td>
<td>b (byte size)</td>
</tr>
<tr>
<td>L (log ave)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Term frequency**
  - n (natural): $t_f, d$
  - l (logarithm): $1 + \log(t_f, d)$
  - a (augmented): $0.5 + \frac{0.5 \times t_f, d}{\max(t_f, d)}$
  - b (boolean): $\begin{cases} 1 & \text{if } t_f, d > 0 \\ 0 & \text{otherwise} \end{cases}$
  - L (log ave): $\frac{1 + \log(t_f, d)}{1 + \log(\text{ave}_{t \in d}(t_f, d))}$

- **Document frequency**
  - n (no): $1$
  - t (idf): $\log \frac{N}{d_f}$

- **Normalization**
  - n (none): $1$
  - c (cosine): $\frac{1}{\sqrt{w_1^2 + w_2^2 + \ldots + w_M^2}}$
  - u (pivoted unique): $1/u$
  - b (byte size): $1/\text{CharLength}^\alpha$, $\alpha < 1$
## Components of tf-idf weighting

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<th>Term frequency</th>
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</thead>
<tbody>
<tr>
<td>n (natural)</td>
<td>tf(_{t,d})</td>
<td>n (no) 1</td>
</tr>
<tr>
<td>l (logarithm)</td>
<td>1 + log(tf(_{t,d}))</td>
<td>t (idf) log(\frac{N}{df_t})</td>
</tr>
<tr>
<td>a (augmented)</td>
<td>0.5 + (\frac{0.5 \times tf_{t,d}}{\text{max}<em>t(tf</em>{t,d})})</td>
<td>p (prob idf) (\max{0, \log \frac{N-df_t}{df_t}})</td>
</tr>
<tr>
<td>b (boolean)</td>
<td>(\begin{cases} 1 &amp; \text{if } tf_{t,d} &gt; 0 \ 0 &amp; \text{otherwise} \end{cases})</td>
<td>u (pivoted unique) (1/u)</td>
</tr>
<tr>
<td>L (log ave)</td>
<td>(\frac{1 + \log(tf_{t,d})}{1 + \log(\text{ave}<em>{t\in d}(tf</em>{t,d}))})</td>
<td>b (byte size) (1/\text{CharLength}^\alpha), (\alpha &lt; 1)</td>
</tr>
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</table>

### Best known combination of weighting options

- Default: no weighting
## Components of tf-idf weighting

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<td>$1/CharLength^\alpha$, $\alpha &lt; 1$</td>
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Default: no weighting
tf-idf example

- We often use different weightings for queries and documents.
tf-idf example

- We often use **different weightings** for queries and documents.
- Notation: qqq.ddd
tf-idf example

- We often use different weightings for queries and documents.
- Notation: qqq.ddd
- Example: ltn.lnc
We often use different weightings for queries and documents.

- Notation: qqq.ddd
- Example: ltn.lnc
- query: logarithmic tf, idf, no normalization

Isn't it bad to not idf-weight the document?

Example query: best car insurance
Example document: car insurance auto insurance
tf-idf example

- We often use **different weightings** for queries and documents.
- Notation: qqq.ddd
- Example: ltn.lnc
- query: logarithmic tf, idf, no normalization
- document: logarithmic tf, no df weighting, cosine normalization
tf-idf example

- We often use **different weightings** for queries and documents.
- Notation: qqq.ddd
- Example: ltn.lnc
- query: logarithmic tf, idf, no normalization
- document: logarithmic tf, no df weighting, cosine normalization
- Isn’t it bad to not idf-weight the document?
tf-idf example

- We often use **different weightings** for queries and documents.
- Notation: qqq.ddd
- Example: ltn.lnc
- query: logarithmic tf, idf, no normalization
- document: logarithmic tf, no df weighting, cosine normalization
- Isn’t it bad to not idf-weight the document?
- Example query: “best car insurance”
We often use different weightings for queries and documents. Notation: qqq.ddd

Example: ltn.lnc

query: logarithmic tf, idf, no normalization
document: logarithmic tf, no df weighting, cosine normalization

Isn’t it bad to not idf-weight the document?

Example query: “best car insurance”
Example document: “car insurance auto insurance”
**tf-idf example: ltn.Inc**

Query: “best car insurance”. Document: “car insurance auto insurance”.

<table>
<thead>
<tr>
<th>word</th>
<th>query</th>
<th>document</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tf-raw</td>
<td>tf-wght</td>
</tr>
<tr>
<td>auto</td>
<td></td>
<td></td>
</tr>
<tr>
<td>best</td>
<td></td>
<td></td>
</tr>
<tr>
<td>car</td>
<td></td>
<td></td>
</tr>
<tr>
<td>insurance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key to columns: tf-raw: raw (unweighted) term frequency, tf-wght: logarithmically weighted term frequency, df: document frequency, idf: inverse document frequency, weight: the final weight of the term in the query or document, n’lized: document weights after cosine normalization, product: the product of final query weight and final document weight.
### tf-idf example: ltn.lnc

Query: “best car insurance”. Document: “car insurance auto insurance”.

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**tf-idf example: ltn.lnc**

Query: “best car insurance”. Document: “car insurance auto insurance”.

<table>
<thead>
<tr>
<th>word</th>
<th>query</th>
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**Scoring**
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The vector space representation of these terms can be calculated using the formula:

\[
\text{product} = \text{query weight} \times \text{document weight}
\]

For the given example:

\[
\text{product} = 0 \times 1 + 0 \times 1 + 1 \times 1 = 1.04
\]

**Questions?**

---

**Scoring**

---

45 von 47
### tf-idf example: ltn.Inc

Query: “best car insurance”. Document: “car insurance auto insurance”.

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\[
\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92 \\
1/1.92 \approx 0.52 \\
1.3/1.92 \approx 0.68
\]
tf-idf example: ltn.Inc

Query: “best car insurance”. Document: “car insurance auto insurance”.

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Final similarity score between query and document: \[ \sum_{i} w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08 \]
### Term frequency tf-idf weighting

#### The vector space

**tf-idf example: ltn.lnc**

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Final similarity score between query and document: \( \sum_i w_{qi} \cdot w_{di} = 0 + 0 + 1.04 + 2.04 = 3.08 \)

Questions?
Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
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- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query
Summary: Ranked retrieval in the vector space model

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity between the query vector and each document vector
- Rank documents with respect to the query
- Return the top $K$ (e.g., $K = 10$) to the user
Resources

- Chapters 6 and 7 of IIR
Resources

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- Resources at http://ifnlp.org/ir
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- Vector space for dummies
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- Exploring the similarity space (Moffat and Zobel, 2005)
- Okapi BM25 (a state-of-the-art weighting method, 11.4.3 of IIR)